AAP China Scholarship Council - CSC 2024

PROJET DE RECHERCHE DOCTORALE (PRD)

Titre du PRD : Geometry of renormalized powers of random fields in dimension 2

DIRECTION de THESE

Porteuse ou porteur du projet (***doit être titulaire de l'HDR***) :

NOM : LUPU

Prénom : Titus

Titre : CR ou Autre :

Section CNU : 26

Email : titus.lupu@sorbonne-universite.fr

Unité de recherche : Code (ex. UMR xxx) et Intitulé : UMR 8001, LPSM

Ecole doctorale de rattachement : ED386 - Sciences Mathématiques de Paris Centre

Nombre de doctorants actuellement encadrés : 0

CO-DIRECTION de THESE (HDR) ou CO-ENCADREMENT (Non HDR) :

NOM :

Prénom :

Titre : Sélectionner ou Autre :

Section CNU :

Email :

Unité de recherche : Code (ex. UMR xxx) et Intitulé :

Ecole doctorale de rattachement Sorbonne Université : Sélectionner ou autre :

Nombre de doctorants actuellement encadrés :

CO-TUTELLE INTERNATIONALE envisagée : ☐ OUI ☒ NON

DESCRIPTIF du PRD :

*Ce texte sera affiché en ligne à destination des candidates et candidats chinois : il ne doit pas excéder **2 pages** doit être rédigé en **ANGLAIS***

Geometry of renormalized powers of random fields in dimension 2

This Doctorate project is in the field of Mathematics, specifically in the domain of Probability Theory, with connections to Geometry in relation to the conformal invariance in dimension 2.

The two central objects of study will be continuum Gaussian free field (GFF) and the Brownian motion, both in dimension 2. The GFF is a random generalized function, not defined pointwise. In dimension 2 it is logarithmically correlated and satisfies a statistical conformal invariance property. It plays a central role in the theory of conformally invariant stochastic processes. Its intrinsic geometry can be described through SLE processes (Schramm-Loewner Evolution). The GFF is also closely related to the Brownian motion (Symanzik, Brydges-Fröhlich-Spencer, Dynkin). In particular, it has been recently shown that the GFF can be decomposed through clusters of Brownian-like loops, where each cluster appears to be a sign connected component of the GFF. The restriction of the GFF to a cluster is, up to the sign, a Minkowski content measure. These clusters can be also described through nested Conformal Loop Ensembles CLE_4.

One of the aims of the Doctorate would be to obtain analogous decompositions for the renormalized (Wick) powers of the GFF in dimension 2. The powers of the GFF are not defined as such, but there is a renormalization procedure involving Hermite polynomials that removes the divergent terms and returns well defined generalized functions, the Wick powers. During the Doctorate one would study the relations between these Wick powers of the GFF and the aforementioned clusters of Brownian loops from a geometrical point of view: how the shape of the clusters and their microscopic properties encode the Wick powers. Since there is a strong analogy between the Wick powers of the GFF and the renormalized self-intersection local times of the planar Brownian motion, these renormalized self-intersection local times would also be a point of study and an important tool.

Besides the Gaussian case, a similar type of questions arises in the study of other confomally invariant random fields, such as the fractional powers of alpha-permanental fields. The extension of the results that would be obtained for the GFF to this more general setting is also a goal of the Doctorate.

Finally, a third related goal would be to understand how the microscopic behaviour of both the clusters of Brownian loops, and that of the random fields and their renormalized powers can be simultaneously approximately described through one-dimensional stochastic processes, such as the 1D Brownian motion and more general Bessel processes. This is due to an asymptotic dimension reduction phenomenon (from 2 to 1), that is yet to be fully understood.

To summarize, the Doctorate will touch to the following topics: Gaussian free field, Wick renormalization, Brownian motion, renormalized self-intersection local times, SLE processes, long range percolation type models, Bessel processes, etc.

The Doctorate applicants must have a training in both probability theory, including stochastic calculus, and in complex analysis. Some background in statistical physics (percolation, etc.) is also recommended.

The planned duration of the Doctorate is 4 years.

AVIS de l'Ecole Doctorale :

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NOM Prénom_Projet CSC 2024.pdf

Fichier à envoyer par mail simultanément
à l'école doctorale de rattachement et à csc-su@listes.upmc.fr