Non-Gaussian quantum correlations and their applications

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Quantum mechanics is at the core of the major development in technology that we witness nowadays. Major efforts are being conducted worldwide to push these new technologies forward to their full potential. These efforts concentrate mainly in the realization of quantum computation and communication protocols, and much is to be expected –and has been attained already– in high precision metrology. Because of their large robustness against decoherence, quantum states of light offer a formidable candidate platform for implementing these quantum technologies. Furthermore, this platform has a good potential for scalability, based on the fact that it can be operated at room temperature, and in its compatibility with readily available communication technologies.

Quantum optical systems, like any bosonic system, can be described in two complementary ways [1]. On the one hand, a discrete-variable approach, that focus on the number of photons as the relevant observable. This approach finds wide applicability as it offers a natural encoding for quantum information, with each photon encoding the state of one qubit. Yet this kind of encoding is highly susceptible to the loss of photons in the optical path, which is the most common source of problems in optical devices, and also suffers from the difficulty of deterministically generating single photons. On the other hand, there is the continuous-variable approach, where the relevant observables are the real and imaginary parts of a mode of the electromagnetic field, called field quadratures. This approach allows to deterministically generate gigantic entangled states of up to one million modes [2], which offer a unique playground for quantum information processing.

The framework of continuous variables allows to describe an infinite-dimensional system within a finite dimensional phase space of two times the number of modes of the system. This description is provided in terms of quasi-probability distributions [1], akin to the classical description of statistical ensembles. Notable within the family of quasi-probability distributions is the Wigner function. Because of the Heisenberg uncertainty principle, that forbids the simultaneous measurement of amplitude and phase quadratures, the Wigner function can reach negative values. Nonetheless, it is the closest to a classical probabilistic description, as it is the only phase space representation of the state that is normalized and marginalizes to the probability density of outcomes of measurements of the corresponding quadrature.

The Wigner function is an important tool to differentiate between Gaussian and non-Gaussian states [1]. By definition, Gaussian states are those that can be described in phase space with a Gaussian Wigner function. These states can be generated and further manipulated with current technology in a deterministic way. In particular, the large multimode entangled states mentioned above are within this family. However, the most exotic quantum features, like Wigner negativity, are found in non-Gaussian states. We need exactly these exotic features, such as negativity in the Wigner function [3] and non-Gaussian entanglement [4] to realize quantum protocols that can not be efficiently simulated with classical resources.

In [4] we showed that the complexity of simulating this optical sampling problems with a classical device is exponential in the non-Gaussianity of the input state, as measured through its stellar rank [5]. Moreover, we provided an efficient algorithm for simulating the sampling for states that can be rendered separable with phase shifters and beamsplitters (passive linear optics). In other words, this result amounts to say that in order to have a sampling problem that is hard to simulate by classical means, the state should exhibit entanglement in every mode basis, because otherwise the setup could be simulated by independently sampling from single mode states. Every Gaussian state can be passively dis-entangled, so that this kind of entanglement can only arise in non-Gaussian states.

This result opens several directions to explore in the formalization of non-Gaussian entanglement and its applications. On the one hand, there are many open questions related to the role of non-Gaussian entanglement in quantum computation. On the other hand, it is a very natural question to wonder whether non-Gaussian entanglement has a particular advantage in quantum metrology.

Description of the project

A) Theoretical framework of non-Gaussian quantum correlations: In order to push forward the understanding of the complexity of quantum computation with continuous-variable systems, a solid theoretical framework for the study of non-Gaussian quantum correlations is missing. The first goal of this project is to build up such a framework and study the properties of non-Gaussian entangled states.
B) Connection between Wigner negativity and non-Gaussian quantum correlations: It is our hope that by building a formal framework for the understanding of non-Gaussian quantum correlations we will shed light on its relation to Wigner negativity. We have reasons to believe such a bridge can be built. The results in [4] naturally complement those of [3], but it is currently not clear how exactly these two results can be connected.
C) Experimental detection of non-Gaussian quantum correlations: The candidate will assist the experimenters when implementing the methods to detect quantum correlations in non-Gaussian states [6] developed recently in the group. These experiments will be done using photon subtracted states, that are currently produced in our lab [7]. Moreover it is a goal of this project to device new methods for the detection of non-Gaussian quantum correlations, informed by the formal framework that we aim to build and by possible connections with Wigner negativity.
D) Metrological advantage through non-Gaussian quantum correlations: The detection methods that we have developed suggest that non-Gaussian features are not only relevant for quantum computation but can also be used as resources for quantum sensing applications. This naturally relates to results linking negativity of quasi-probability distributions and quantum metrological advantage [8]. Through this connection, we expect to be able to clarify the relation between non-Gaussian quantum correlations and metrological advantage.

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