

Sorbonne Université/China Scholarship Council program 2021

Thesis proposal

Title of the research project: On the Langlands correspondence for non-supercuspidal representations of p -adic reductive groups.

Keywords: Langlands correspondence, non archimedean field, p -adic group, representation, local shtuka, Fargues-Fontaine curve

Joint supervision: no

Joint PhD (cotutelle): no

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Doctoral school (N° +name): ED 386 École Doctorale de Sciences Mathématiques de Paris-Centre.

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Subject description (2 pages max):

1) Study context

The Langlands Program proposes a non-abelian generalization of Artin's reciprocity laws, by building bridges between two general areas of pure Mathematics, namely, Number Theory and Representation Theory. It also touches upon more specialized fields of modern research, for example, Arithmetic Geometry and Automorphic Forms. This

intricate field of active research is structured by a vast set of conjectures, reciprocity and functoriality, that first appeared in a Langlands' letter to André Weil in 1967 (see [A] for more details). The subject of the thesis is concerned with **local** aspects of this program.

Let G be a connected reductive group over a p -adic field E (i.e., a finite extension of the p -adic numbers field) and let $G(E)$ denote the E -points of G . The **local Langlands correspondence** for $G(E)$ asserts that there should be a **natural** surjection with finite fibers from the set $\Pi(G)$ of irreducible smooth representations of the p -adic group $G(E)$ to the set $\Phi(G)$ of Langlands parameters for $G(E)$, which are certain analogues of Galois representations. The existence of the correspondence has been established when $G = \mathrm{GL}_n$ independently by Harris and Taylor, by Henniart and by Scholze, and more recently when G is a classical group by Arthur.

When G is any split connected reductive group over E , and p is not too small, we have constructed in [ABPS] a surjection \mathcal{L} from the subset of $\Pi(G)$ formed by the irreducible representations which occur in the principal series of $G(E)$ to a certain well-defined subset of $\Phi(G)$, that satisfies many of the expected properties of the local Langlands correspondence. Thanks to the main result of [AMS], this construction extends to an analogous map \mathcal{L} for any quasi-split group G .

The subject of the thesis is to construct a geometrization of \mathcal{L} .

2) Details of the proposal

The study of the local Langlands correspondence motivated the theory of *local Shimura varieties*, originally formulated by Rapoport and Viehmann in [RV], with the expectation that the l -adic cohomology of local Shimura varieties should realize many cases of the local Langlands correspondence. For example, the proof by Harris and Taylor of the correspondence for GL_n uses the l -adic cohomology of the Lubin-Tate space, which can be regarded as a local Shimura variety for GL_n .

Let G be a quasi-split connected reductive group over E . For the representations in the principal series of $G(E)$ (or more generally, non-supercuspidal representations), an important conjecture, namely the Harris-Viehmann conjecture, relates the l -adic cohomology of local Shimura varieties to the local Langland correspondence. The heart of

the proposal will be the study of this conjecture in the case of representations in the principal series of $G(E)$.

The first step towards a proof of the conjecture will be to understand certain local Shimura varieties with emphasis on their relation to the local Langlands correspondence. Let E' denote the p -adic completion of the maximal unramified extension of E and let $\sigma \in \text{Gal}(E'/E)$ be the relative Frobenius automorphism.

A *local Shimura datum* is a triple $(G, [b], \{\mu\})$ consisting of a connected reductive group G over E , a σ -conjugacy class $[b]$ of $G(E')$ (as defined in [K]), and a geometric conjugacy class $\{\mu\}$ of cocharacters satisfying certain axioms.

The local Shimura variety associated to a datum $(G, [b], \{\mu\})$ is a certain tower of rigid analytic spaces over E' . Under some conditions, it arises from a moduli space of p -divisible groups, known as a *Rapoport-Zink space*. Roughly speaking, the Harris-Viehmann conjecture says that when the underlying « local shtuka datum » is not « basic », the (class in the Grothendieck group of the) cohomology of this space is parabolically induced (see [Har] and [RV]).

Until rather recently the Rapoport-Zink spaces were the most general available family of moduli spaces of p -divisible groups. In his 2014 course in Berkeley [SW], Scholze vastly generalized the above ideas by constructing moduli spaces of mixed-characteristic local shtukas, which can be viewed as infinite-level Rapoport-Zink spaces. The second step of the proposal will be to reformulate the Harris-Viehmann conjecture as a conjecture on the cohomology of these new moduli spaces using the tools recently developed by Fargues and Scholze (see [F]) and to prove it, by extending to G the strategy used by Hansen in the case of GL_n in [Han].

The next step will be to relate the above construction to the parametrization established in [ABPS] and [AMS] in the case of representations in the principal series of $G(E)$ and to geometrize it in the spirit of [F].

3) References

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[SW] P. Scholze and J. Weinstein, Berkeley lectures on p-adic geometry, Annal of Math Studies, 207, to appear.

4°) Profile of the Applicant (skills/diploma...)

The applicant is YU Jiahong:

Date of birth: 15/03/1997, place of birth: Dalian/Liaoning/China, Nationality: Chinese.

BSc. in Pure Mathematics, School of Mathematical Sciences, Peking University.

OverallGPA:3.425/4.0,MajorGrade:3.82/4.0 (90/100)

Ranking: rank2 among 34 pers. in the department.

Jiahong Yu is an extremely brilliant and very motivated student. His learning and exploration in number theory have been developing at Peking University since 2015.

He is doing in 2020-2021 a Master 2 in Mathematics at the Université de Paris, thanks to a grant from the Paris Graduate School of Mathematical Sciences (PGSM) of the Fondation Sciences Mathématiques de Paris (FSMP). He will do his Master 2 Memoir under my supervision.

He follows regularly students seminars (notably a running seminar organized by Chuan Qin, Ph. D student at Sorbonne Université, on the reference [Har], and a Londonian seminar on Fargues-Scholze's work), and had given numerous talks. He is also following the course given online by Scholze.

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Thesis supervisor

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